The papers /10, 13/ are devoted to an investigation of soliton solutions for the model equation, and the paper /14/ is devoted to non-stationary solutions of solitary wave type.

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INFLUENCE OF NARROW CYLINDRICAL CAVITIES ON THE WAVE FIELD EXCITED BY A CONCENTRATED FORCE IN AN ELASTIC SPACE*

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An elasticity theory problem is considered concerning the excitation of a wave field in a space weakened by a system of cylindircal cavities of small radius with rigid walls, with a concentrated force applied to a certain point of the space outside the shafts and varying sinusoidally. The solution of this problem is constructed by the principle of superposing the solutions of the following problems: the non-axisymmetric vibrations of an elastic space subjected to an oscillating concentrated force (problem 1); the wave field that occurs in an elastic space perforated by a system of narrow cavities vibrating under the effect of a sinusoidally varying stress applied to their walls (problem 2).

We also apply the method elucidated below to the investigation of the displacement field in an elastic space equipped with a system of elastic cylindrical inclusions of small diameter, or a system of cavities filled with liquid or a viscoelastic medium.

1. We consider problem 1. We obtain formulas describing the wave field in a space excited by a concentrated force $\mathbf{X}e^{-i\omega t}$ ($\mathbf{X} = \{X_1, X_2, X_3\}$, ω is the vibration frequency) applied to a

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point of space with cylindrical coordinates r_0, φ_0, z_0 by the method of integral transforms, using the radiation principle /1/. The amplitude values of the displacement vector components $\mathbf{u}^* = \{u^*, v^*, w^*\}$ have the following form in cylindrical coordinates:

$$u^*(r, \varphi, z) = \varkappa_3 \int_\sigma e^{i\alpha(z-z_0)} \sum_{m=1,2} (-1)^m \{[K_{0m} \cos\varphi - K_{2m} \cos(2\psi + \varphi)] X_1 + \tag{1.1}$$

$$[K_{0m} \sin\varphi + K_{2m} \sin(2\psi + \varphi)] X_2 - 2K_{1m} \cos(\psi - \varphi) X_3\} d\alpha$$

$$v^*(r, \varphi, z) = \varkappa_3 \int_\sigma e^{i\alpha(z-z_0)} \sum_{m=1,2} (-1)^m \{[K_{2m} \sin(2\psi + \varphi) - K_{0m} \sin\varphi] X_1 +$$

$$[K_{0m} \cos\varphi + K_{2m} \cos(2\psi + \varphi)] X_2 - 2K_{1m} \sin(\psi - \varphi) X_3\} d\alpha$$

$$w^*(r, \varphi, z) = \varkappa_3 \int_\sigma e^{i\alpha(z-z_0)} \sum_{m=1,2} (-1)^m \{K_{1m} (\cos\varphi X_1 + \sin\psi X_2) +$$

$$(\delta_{2m} \varkappa_2^2 - \alpha) K_0 (R\sigma_m) X_3\} d\alpha$$

$$K_{0m} = [\alpha^2 + (-1)^m \varkappa_2^2] K_0 (R\sigma_m), \quad K_{1m} = i\alpha\sigma_m K_1 (R\sigma_m)$$

$$K_{2m} = \sigma_m^2 K_2 (R\sigma_m), \quad \sigma_m = (\alpha^2 - \varkappa_m^2)^{1/2}, \quad m = 1, 2$$

$$\varkappa_1^2 = \rho\omega^2/(\lambda + 2\mu), \quad \varkappa_2^2 = \rho\omega^2/\mu, \quad \varkappa_3 = -1/(2\pi\varkappa_2^2)$$

$$e^{i\psi} = -r/R e^{i(\varphi-\varphi_0)} + r_0/R, \quad R = [r_0^2 + r^2 - 2r_0 r \cos(\varphi - \varphi_0)]^{1/2}$$

Here ρ is the density of the elastic medium, λ, μ are the Lamé constants, δ_{lm} is the Kronecker delta, and $K_n(x)$ is the Macdonald function. The relationships (1.1) are written in dimensionless form, the displacements are referred to a linear unit, and the forces to the shear modulus μ. The contour of integration σ is selected in accordance with the radiation conditions /1/. It is on the real axis, deviates from it in the positive half-plane, bypasses the bifurcation points $-\varkappa_1, -\varkappa_2$, of the integrand, and bypasses the bifurcation points $\varkappa_1, \varkappa_2$ in the negative half-plane.

The Fourier transforms of the displacement vector components on a cylindrical surface S of small radius a with central axis parallel to the z axis and passing through the point (b, γ, z_0) are determined by the formulas

$$u^*(r, \varphi, z) = \sum_{p=-\infty}^{\infty} u^*(r, p, z) e^{ip\varphi} \tag{1.2}$$

$$U^*(r, p, \alpha) = \int_{-\infty}^{\infty} u^*(r, p, z) e^{-i\alpha z} dz \tag{1.3}$$

$$U^*(b, \pm 1, \alpha)|_{r,\varphi\in S} = \varkappa_3 e^{-i\alpha z_0} \sum_{m=1,2} (-1)^m [(K_{0m} - K_{2m} e^{\pm 2i\theta} X_1) \mp \tag{1.4}$$

$$i(K_{0m} + K_{2m} e^{\pm 2i\theta}) X_2 - 2K_{1m} e^{\mp i\theta} X_3]$$

$$V^*(b, \pm 1, \alpha)|_{r,\varphi\in S} = \pm iU^*(b, \pm 1, \alpha)|_{r,\varphi\in S}$$

$$W^*(b, 0, \alpha)|_{r,\varphi\in S} = \varkappa_3 e^{-i\alpha z_0} \sum_{m=1,2} (-1)^m [K_{1m} (\cos\theta X_1 + \sin\theta X_2) +$$

$$(\delta_{2m}\varkappa_2^2 - \alpha) K_0 (R\sigma_m)], \quad e^{i\theta} = -b e^{i(\gamma-\varphi_0)}/R + bR$$

$$W^*(b, \pm p, \alpha)|_{r,\varphi\in S} = O(a^p), \quad p \geqslant 1$$

$$U^*(b, 0, \alpha)|_{r,\varphi\in S} = O(a), \quad U^*(b, \pm p, \alpha)|_{r,\varphi\in S} = O(a^{p-1}), \quad p \geqslant 1$$

$$V^*(b, 0, \alpha)_{r,\varphi\in S} = O(a), \quad V^*(b, \pm p, \alpha)|_{r,\varphi\in S} = O(a^{p-1}), \quad p \geqslant 1$$

Formulas (1.4) are derived from (1.1) by using the addition theorem for modified Bessel functions and their asymptotic forms for small argument /2/.

2. Problem 2 is described by the Lamé equation in a cylindrical coordinate system /3/ and the boundary conditions

$$\mathbf{q}(r, \varphi, z) e^{-i\omega t}|_{r,\varphi\in S_j} = \mathbf{q}_j(a, \varphi, z) e^{-i\omega t}, \quad j = 1, 2, \ldots, N \tag{2.1}$$

where $\mathbf{q}_j(a, p, z) = \{q_j, p_j, \tau_j\}$ is the amplitude value of the stress vector on the side surface S_j of of the j-th cavity, $a \ll 1$ is the cavity radius, and N is the number of cavities in the system. The cavity generators and the z axis are parallel. The position of the cavities in the system is determined by the distance between the centres of the i-th and j-th cavity b_{ij} and by the angle between the polar axis and the perpendicular connecting the centres of the cavities γ_{ij}.

Since the vibrations regime is assumed steady, we shall in the sequel use just the amplitude values of the corresponding functions.

Applying the Fourier integral transform to the Lame equations and satisfying the conditions (2.1), we arrive at formulas describing the displacement field $\mathbf{u} = \{u, v, w\}$

$$\mathbf{u}(r, \varphi, z) = \sum_{p=-\infty}^{\infty} e^{ip\varphi} \int_\sigma e^{i\alpha z} K(r, p, \alpha, a) \mathbf{Q}_1(a, p, \alpha) d\alpha + \tag{2.2}$$

$$\sum_{j=2}^{N} \sum_{m=-\infty}^{\infty} e^{im\psi} \int_\sigma e^{i\alpha z} K(\rho_j, m, \alpha, a) \mathbf{Q}_j(a, m, \alpha) d\alpha$$

$$\mathbf{q}(a,\varphi,z)=\sum_{p=-\infty}^{\infty}\mathbf{q}(a,p,z)e^{ip\varphi} \tag{2.3}$$

$$\mathbf{Q}_j(a,p,\alpha)=\int_{-\infty}^{\infty}e^{-i\alpha z}\mathbf{q}_j(a,p,z)\,dz$$

$$\begin{aligned}
&\rho_j=(r^2+b_{1j}^2-2rb_{1j}\cos\varphi)^{1/2},\quad e^{\pm i\psi}=(re^{\pm i\varphi}-b_{1j})/\rho_j\\
&K(r,p,\alpha,a)=F(r,p,\alpha)\,C(a,p,\alpha)\\
&F(r,p,\alpha)=\{F_{ij}\},\ i,j=1,2,3\\
&F_{11}=\partial R_{p1}/\partial r,\ F_{21}=ip/rR_{p1},\ F_{31}=-i\alpha R_{p1}\\
&F_{1j}=\alpha mR_{(p+m)2},\ F_{2j}=-imF_{1j},\ F_{33}=im\sigma_2R_{p2},\ m=(-1)^j,\ j=1,2\\
&C^{-1}(a,p,\alpha)=\{G_{ij}\}|_{r=a},\ i,j=1,2,3\\
&G_{11}=2\partial^2R_{p1}/\partial r^2-(\lambda+\mu)\varkappa_1^2R_{p1}/\mu;\ G_{21}=-2i\alpha\partial R_{p1}/\partial r\\
&G_{31}=2ip/r\,(\partial R_{p1}/\partial r-R_{p1}/r);\ G_{1j}=2m\alpha\partial R_{(p+m)2}/\partial r\\
&G_{2j}=im\,[mp\sigma_2/rR_{p2}-(\alpha^2+\sigma_2^2)R_{(p+m)2}]\\
&G_{3j}=i\alpha\sigma_2R_{(p+2m)2},\ j=2,3;\ R_{mn}=K_m(\sigma_nr),\ n=1,2
\end{aligned} \tag{2.4}$$

For small a and $p>1$ the elements of the matrix $C(a,p,\alpha)$ have the following asymptotic behaviour

$$\begin{aligned}
&C_{i2}(a,p,\alpha)=O\left(\frac{a^{p+1}}{p!2^{p-1}}\right);\quad i=1,2,3\\
&C_{ij}(a,p,\alpha)=O\left(\frac{a^p}{p!2^{p-1}}\right);\quad j=1,3,\quad i=1,2,3
\end{aligned} \tag{2.5}$$

On the basis of (2.4) and (2.5), the terms of the infinite series in (2.2) decrease as $a^p/2^{p-1}p!$, hence, it is sufficient to consider just the first harmonics $p=0,\pm1$, here the elements of the matrices $K(r,0,\alpha,a)=\{K_{ij}^\circ\}$, $K(r,\pm1,\alpha,a)=\{K_{ij}^{\pm1}\}$ have the simple form

$$\begin{aligned}
&K_{11}^\circ=K_{12}^\circ=K_{21}^\circ=K_{23}^\circ=K_{31}^\circ=K_{32}^\circ=K_{22}^\circ=0\\
&K_{13}^\circ=i\alpha a\,(\sigma_1R_{11}-\sigma_2R_{12})/\varkappa_2^2,\ K_{33}^\circ=a\,(\sigma_2^2R_{02}-\alpha^2R_{01})/\varkappa_2^2\\
&K_{11}^{\pm1}=-a\,[2\sigma_1\partial R_{11}/\partial r+\sigma_2^2R_{22}+(\alpha^2+\varkappa_2^2)R_{02}]/(4\varkappa_2^2)\\
&K_{21}^{\pm1}=-ai\,[2\sigma_1R_{11}/r-\sigma_2^2R_{22}+(\alpha^2+\varkappa_2^2)R_{02}]/(4\varkappa_2^2)\\
&K_{31}^{\pm1}=0.25K_{13}^\circ;\ K_{13}^{\pm1}=K_{23}^{\pm1}=K_{33}^{\pm1}=0\\
&K_{12}^{\pm1}=\mp iK_{11}^{\pm1};\ K_{22}^{\pm1}=\mp iK_{21}^{\pm1};\ K_{32}^{\pm1}=\mp iK_{31}^{\pm1}
\end{aligned} \tag{2.6}$$

3. On the basis of the solutions of problems 1 and 2, there is the wave field

$$\mathbf{u}^\circ(r,\varphi,z)=\mathbf{u}(r,\varphi,z)+\mathbf{u}^*(r,\varphi,z) \tag{3.1}$$

The origin is on the axis of symmetry of one of the cavities; we shall consider it as cavity No.1.

The unknown functions $\mathbf{Q}_i(a,p,\alpha)$, $i=1,2,\ldots,N$ in (3.1) that determine the stresses on the cavity walls are found from the cavity wall stiffness conditions

$$\mathbf{u}^\circ(r,\varphi,z)|_{r,\varphi\in S_j}=0,\ j=1,2,\ldots,N \tag{3.2}$$

We will satisfy conditions (3.2) for each harmonic of the Fourier series of the displacements, we first pass from the local coordinate system ρ_j,ψ_j with origin at the centre of the i-th cavity to the r,φ coordinate system in (2.2) by using the addition theorem for Macdonald functions /2/. Taking account of the smallness of the cavity radius a, the formulas and asymptotic estimates (1.4), and neglecting terms whose contribution is small compared with the rest, we arrive at a system of integral equations of the second kind to determine the unknown functions $\tau_j(a,0,z)$ and the combinations $q_j^1=q_j(a,1,z)-ip_j(a,1,z)$; $q_j^2=q_j(a,-1,z)+ip_j(a,-1,z)$; the system has the following form in Fourier transforms

$$\begin{aligned}
&\ln a\mathrm{T}_j(a,0,\alpha)-\varkappa_2^{-2}\sum_{\substack{m=1\\m\neq j}}^{N}\{y_1(\alpha,b_{jm})\mathrm{T}_m(a,0,\alpha)+i/2\alpha y_2(\alpha,b_{jm})\times\\
&\quad[Q_m^2\exp(-i\gamma_{jm})-Q_m^1\exp(i\gamma_{jm})]\}=a^{-1}W^*(b_{1j},0,\alpha)|_{r,\varphi\in S_j}\\
&(\varkappa_1^2+\varkappa_2^2)a\ln aQ_j^1+\sum_{\substack{m=1\\m\neq j}}^{N}\{i\alpha\exp(-i\gamma_{jm})y_2(\alpha,b_{jm})\mathrm{T}_m(a,0,\alpha)+\\
&\quad y_3(\alpha,b_{jm})Q_m^1+\exp(-2i\gamma_{jm})y_4(\alpha,b_{jm})Q_m^2\}=-4\varkappa_2^2U^*(b_{1j},1,\alpha)|_{r,\varphi\in S_j}\\
&(\varkappa_1^2+\varkappa_2^2)a\ln aQ_j^2+\sum_{\substack{m=1\\m\neq j}}^{N}\{i\alpha\exp(i\gamma_{jm})y_2(\alpha,b_{jm})\mathrm{T}_m(a,0,\alpha)+\\
&\quad\exp(2i\gamma_{jm})y_3(\alpha,b_{jm})Q_m^1+y_4(\alpha,b_{jm})Q_m^2\}=-4U^*(b_{1j},-1,\alpha)|_{r,\varphi\in S_j}\\
&y_1(\alpha,r)=\alpha^2R_{01}-\sigma_2^2R_{02},\ y_2(\alpha,r)=\sigma_1R_{11}-\sigma_2R_{12}\\
&y_3(\alpha,r)=\sigma_1^2R_{21}-(\alpha^2+\varkappa_2^2)R_{22},\ y_4(\alpha,r)=\sigma_1^2R_{01}-\sigma_2^2R_{02}
\end{aligned} \tag{3.3}$$

The relationship between the quantities denoted by the upper and lower case letters q, p, τ, is given by (2.3).

System (3.3) can be solved by successive approximations. It is easy to see that its order is reduced to 3.

The mutual influence of the narrow cavities is traced only in the lower harmonics $p = 0, \pm 1$ of the Fourier-series expansions; starting with $p \geqslant 2$ it is negligible. The presence of adjacent cavities exerts no influence on the determination of the functions

$$Q_j(a, 0, \alpha), \quad P_j(a, 0, \alpha), \quad \tau_j(a, \pm 1, \alpha), \quad Q_j(a, \pm p, \alpha), \ p \geqslant 2$$

These functions introduce a contribution of the order of a^2 and higher to the wave field. The functions $\tau_j(a, 0, \alpha), Q_j^1, Q_j^2$ depend on the mutual location of the cavities and the stresses on their walls. The wave-field component due to these functions is of the order of $1/\ln a$. Taking account of (2.5) and the above, we arrive at the conclusion that the wave field of the problem is determined by the functions $\tau_j(a, 0, \alpha), Q_j^1, Q_j^2$. The displacement field of the initial problem is determined by (3.1), (2.2), (1.1); it is sufficient to set $p, m = 0, \pm 1$ in (2.2)

In the case $N = 1$, relationships (3.1) take the simplest form

$$u^\circ(r, \varphi, z) = \frac{1}{\ln a} \int_\sigma e^{i\alpha z} \Big\{ -\frac{i\alpha}{\varkappa_2^2} y_4(a, z) W^*(a, 0, a) +$$

$$\frac{K_{11}^1(r, 1, \alpha, a)}{\varkappa_1^2 + \varkappa_2^2} [e^{i\varphi} U^*(a, 1, \alpha) + e^{-i\varphi} U(a, -1, \alpha)] \Big\} d\alpha + u^*(r, \varphi, z)$$

$$v^\circ(r, \varphi, z) = \frac{1}{\ln a\, (\varkappa_1^2 + \varkappa_2^2)} \int_\sigma e^{i\alpha z} K_{21}^1(r, 1, \alpha, a) [e^{i\varphi} U^*(a, 1, \alpha) + e^{-i\varphi} U^*(a, -1, \alpha)]\, d\alpha + v^*(r, \varphi, z)$$

$$w^\circ(r, \varphi, z) = -\frac{1}{\ln a} \int_\sigma \varkappa_2^{-2} e^{i\alpha z} \Big\{ y_1(\alpha, r) W^*(a, 0, \alpha) +$$

$$\frac{i\alpha}{\varkappa_1^2 + \varkappa_2^2} y_2(\alpha, r) [e^{i\varphi} U^*(a, 1, \alpha) + e^{-i\varphi} U^*(a, -1, \alpha)] \Big\} da + w^*(r, \varphi, z)$$

It should be noted that near the side surface of the cavities the displacement field components caused by the influence of the adjacent cavities and the cavity itself are related as $1 : \ln a$.

A system of cavities of small radius a that perforates an elastic space causes a perturbation of the order of $1/\ln a$ in the wave field of an elastic medium. This contribution depends on both the number of cavities in the system and on their arrangement.

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THE POSSIBILITY OF IRREVERSIBLE QUASISTATIC PROCESSES IN A MACROSYSTEM*

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For a fairly long time only such systems for which the running macrostate of the system is practically independent of the preceding history of change in the external parameters for quasistatic * (The definition of a quasistatic process used here corresponds to the standard definition (for instance, /1—5/), while the reversibility of the process is understood in the narrow sense /5/.) processes, have been considered in thermodynamics. The absence of a "memory" is characteristic for quasistatic processes in gases and ordinary liquids, as for reversible processes in any system.

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